



York Region District School Board

**Student Work Study:
Learning Through the Work
in Mathematics**

**Deepening the Understanding of the
Student Learning Experience**

June 2014

A Collaborative Learning Monograph

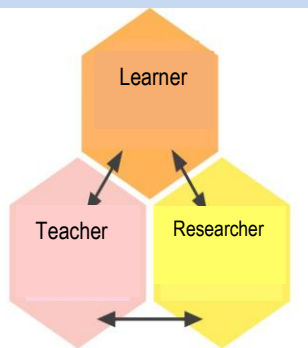
**Joan Bullivant
Nancy Golding
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YRDSB
SWS Teacher Researchers**

PURPOSES of the SWS Teacher Researcher Role:

The work of the SWS Teacher Researcher has three interrelated purposes:

- To document and study student actions in mathematical contexts in order to inform potential actions
- To work through co-learning relationships with students, host teachers and schools focused on student learning
- To create and share a research-based monograph.

Learning Through the Work (LNS, Fall 2012)



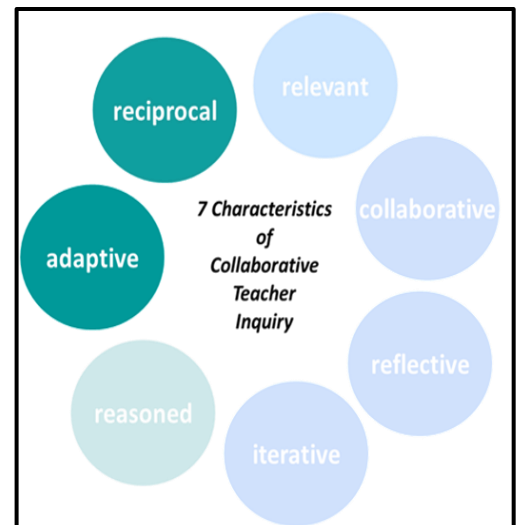
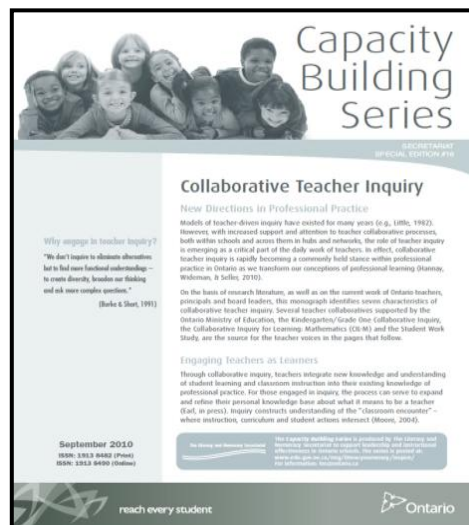
The Student Work Study (SWS) Initiative is a provincial study established and funded by the Literacy and Numeracy Secretariat (LNS). The SWS Initiative enables provincial school districts and LNS to work in partnership to improve student learning and achievement in both literacy and numeracy for students from K – 6. The SWS Initiative engages teachers as co-learners in a collaborative teacher inquiry.

In February 2011, the SWS Initiative started in York Region District School Board (YRDSB) with four SWS teachers (SWST) observing students in their classrooms with a focus on moving students to the provincial standard and beyond in literacy or mathematics. The SWSTs worked collaboratively with host teachers endeavouring to learn more about the characteristics of student work and the classroom conditions which moved student thinking and learning forward.

In September 2012, there were five SWSTs working throughout the district. Each SWST was assigned to a different Community Education Centre (CEC) representing a geographical area in our district, with one SWST working in a Full-Day Kindergarten designated school in each area.

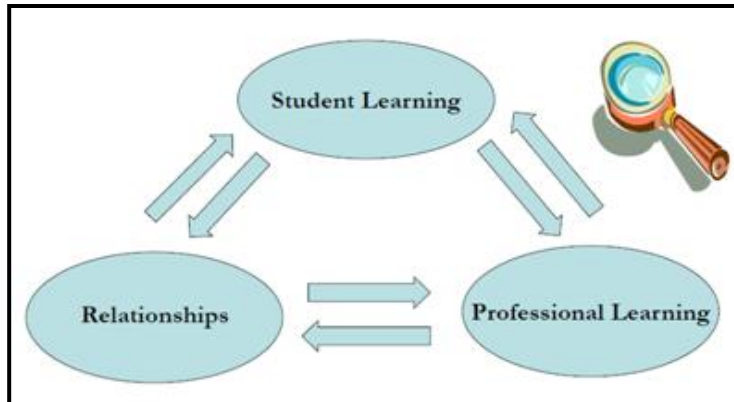
At the onset of this school year 2013 - 2014, it was determined that the focus of the study was mathematics. There were three SWSTs assigned to four schools each throughout the district. The host schools were selected according to “measures of need” in mathematics.

The SWS approach in all of the host classrooms is based on the collaborative teacher inquiry model as outlined in the Collaborative Teacher Inquiry Monograph (Capacity Building Series, #16, 2010).



The seven characteristics of Collaborative Teacher Inquiry guide the work of the teacher researchers and host teachers.

In essence, we take an **open-to-learning stance** which means we try to enter into classrooms without pre-conceived ideas or judgments. Instead we value the ideas and thoughts of all the participants. Our conversations with our colleagues are anchored in student work. We use the students' perspective via their voice, actions or thinking to enter into reflective discussions about the task, the curriculum (the content) and related pedagogical knowledge which may result in a shift in thinking for all the participants.



Developing trusting relationships is paramount. Positioning ourselves as learning partners as we co-inquire about student learning is crucial. With improved student learning as our focus, we enter into relationships of trust with our host teachers in an effort to build our collective capacity: “Inquiry positions the teacher as an informed practitioner refining planning, instruction and assessment approaches in the continual pursuit of greater precision, personalization and innovation” (CBS, #16, 2010, p. 2).

COMBINED PARTICIPATION DATA

October 2013 - June 2014

| Student Work Study Teacher | Total No. Of Host Teachers | Total No. Of Invited Participants | FDK | Primary | Junior | Intermediate | Literacy Inquiries | Numeracy Inquiries |
|----------------------------|----------------------------|-----------------------------------|-----|---------|--------|--------------|----------------------------------|--------------------|
| SWST #1 | 15 | 8 | 3 | 6 | 6 | 0 | 2 (Oral Language Development) | 13 |
| SWST #2 | 15 | 8 | 3 | 4 | 8 | 1 | 0 | 15 |
| SWST #3 | 16 | 12 | 1 | 10 | 5 | 0 | 0 | 16 |
| Total | 46 | 28 | 7 | 20 | 19 | 1 | 2 | 44 |

SUPPOSITIONAL STATEMENT OF INQUIRIES

When students can communicate and represent their thinking both orally and visually when solving problems, they are better able to construct deeper mathematical understandings.

*The purpose of this study is to document student actions in the classroom to better understand the student learning experience and to improve student achievement in mathematics. Classroom teachers from participating host schools collaborated with the SWS teacher researcher to inquire into student learning. Through the iterative cycle of collaborative inquiry, documentation in the form of digital recordings of student conversations and interactions along with student work were analyzed to determine student strengths and most urgent learning needs which informed next steps for instruction. Collective analysis of data revealed that students were struggling to communicate their thinking both orally and in written form and to represent their thinking using various models when solving problems. The students were experiencing difficulty initiating the problem-solving process, initiating and sustaining a productive math conversation, persevering to solve the problem, and making their thinking visible in order to self-regulate. Professional learning opportunities referencing current research informed the study and determined next steps for instruction. This study was limited in the fact that documentation and immediate follow-up were not always possible as visits to host schools occurred once a week. Conclusions contribute to the theory that when students make their thinking visible, they are better able to communicate and represent their constructed conceptual understandings. Implications of this finding demand teacher and student attention to the mathematical processes found in *The Ontario Curriculum, Grades 1-8, Mathematics (2005)*. Embedding these processes throughout the mathematics program allows students to communicate, represent and reflect upon their solutions when solving problems while constructing mathematical understandings. Further analysis involving the application of the mathematical processes and their impact upon student learning are summarized in a *Collaborative Learning Monograph June 2014 informed by individual studies conducted by the SWS teacher researcher team*.*

Primary Resources Consulted:

Chapin, Suzanne H. & Catherine O'Connor & Nancy Canavan Anderson. (2009). *Classroom Discussions: Using math talk to help students learn, Grades K – 6 (Second Edition)*. Sausalito, CA: Math Solutions.

Hull, Ted H. & Don S. Balka & Ruth Harbin Miles. (2011). *Visible Thinking in the K – 8 Mathematics Classroom*. Thousand Oaks, CA: Corwin A SAGE Company.

Ontario Ministry of Education. (2005) *The Ontario Curriculum, Grades 1 – 8, Mathematics (Revised)*. Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education. (2006) *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, Volume Two, Problem Solving and Communication*. Toronto, ON: Queen's Printer for Ontario.

York Region District School Board. BWW: *Mathematical Literacy Resources*. Retrieved June 2014. <https://bww.yrdsb.ca/services/cis/mathliteracy/Pages/default.aspx>

When students construct mental math models, then their understanding of number relationships is deepened.

The purpose of this study is to document student actions in the classroom to better understand the student learning experience and to improve student achievement in mathematics. Observations of student learning were collected in four host schools across 15 host classrooms. Digital recordings of student conversations and actions and anecdotal records, collected during weekly encounters with students, were analyzed to identify students' strengths and gaps in students' learning. Collective analysis of student evidence was used as an improvement tool to guide collaboration and inform the next steps for instruction. Evidence of student learning revealed that some students were not yet using relationships between numbers as shortcuts to automatize facts or to develop flexible strategies for computing with numbers. Research based resources and strategies informed teaching moves to support students in constructing concrete and pictorial models to help them develop an understanding of quantity and operational sense. Limitations included gaps in classroom visits. Observations of student learning and follow-up conversations with host teachers were limited to weekly visits. Opportunities to observe the classroom experience and shifts in students' thinking on a daily basis were not possible. Conclusions support the theory that when students construct mental math models, then their understanding of number relationships is deepened. If we want students to work mathematically in flexible, efficient and innovative ways, then daily attention to number sense and numeration must be made a priority. Students need a variety of experiences with quantity and operations, modeling by teachers and peers of student thinking and time to construct mental math models to deepen their understanding of relationships between numbers. Further analysis involving the application of the mathematical processes and their impact upon student learning are summarized in a Collaborative Learning Monograph June 2014 informed by individual studies conducted by the SWS teacher researcher team.

Primary Resources Consulted:

Fosnot, Catherine Twomey and Dolk, Maarten. (2007) Context for Learning. (Portsmouth, N.H.: Heinemann).

Fosnot, Catherine Twomey. (2010). Models of Intervention in Mathematics Reweaving the Tapestry. (USA: National Council of Teachers of Mathematics/Pearson).

Fosnot, Catherine Twomey and Dolk, Maarten. (2001) Young Mathematicians at Work Series. (Portsmouth, N.H.: Heinemann).

Ontario Ministry of Education.(2005) The Ontario Curriculum, Grades 1-8, Mathematics. Toronto: Queen's Printer of Ontario.

Ontario Ministry of Education. (2003) A Guide to Effective Instruction in Mathematics K-6, Toronto: Queen's Printer of Ontario

Sousa, David A.. (2008). How the Brain Learns Mathematics. (California: Corwin Press).

York Region Board of Education. Mathematical Literacy Resources. Retrieved June 2014.
<https://www.yrdsb.ca/services/cis/mathliteracy/Pages/default.aspx>

When students build a personalized repertoire of strategies and models they can draw from, their ability to solve mathematical problems is enhanced.

The purpose of this study is to document student actions in the classroom to better understand the student learning experience and to improve student achievement in mathematics. Classroom teachers from participating host schools collaborated with the SWS teacher researcher to inquire into student learning. Through the iterative cycle of collaborative inquiry, documentation in the form of observations, conversations and student work were analyzed to determine student strengths and the most urgent learning needs which informed next steps for instruction. Collective analysis of data revealed that students were unsure about how to solve problems. They were unsure how to enter problems, choose strategies, represent their thinking through models, or solve problems efficiently. Professional learning opportunities referencing current research informed this study and determined next steps for instruction. This study was limited in the fact that daily documentation with immediate follow-up were not possible. Conclusions contribute to the theory that when students build a personalized repertoire of strategies and models they can draw from, their ability to solve mathematical problems is enhanced. Implications of this finding demand teacher and student attention to the mathematical processes found in the Ontario Mathematics Curriculum document. Embedding these processes throughout the mathematics program allows students to demonstrate and articulate their solutions when solving problems. This is explored further in the Collaborative Learning Monograph which denotes a cross-case analysis of 2013 - 2014 YRDSB Learning Monographs by individual SWS teacher researchers.

Primary Resources Consulted:

Fosnot, Catherine Twomey and Dolk, Maarten. (2001) Young Mathematicians at Work Constructing Number Sense, Addition, and Subtraction. Portsmouth, N.H.: Heinemann.

Fosnot, Catherine Twomey. (2010). Models of Intervention in Mathematics Reweaving the Tapestry. USA: National Council of Teachers of Mathematics/Pearson.

Hull, Ted H and Balka, Don S. and Harbin Miles, Ruth. (2011). Visible Thinking in the K-8 Mathematics Classroom. (2011) California: Corwin/NCTM.

Ministry of Education. (2005). The Ontario Curriculum, Grades 1-8: Mathematics (revised). Toronto: Queens Printer for Ontario.

Ministry of Education. (2006b). A Guide to Effective Instruction in Mathematics Kindergarten to Grade 6. Queens Printer for Ontario

York Region District School Board. BWW: Mathematical Literacy Resources.
Retrieved June 2014. <https://yrdsb.ca/services/cis/mathliteracy/Pages/default.aspx>

SETTING THE CONTEXT: Global

Student engagement and curiosity could be addressed through stronger development of 21st Century learning skills and well-being. We could call this the 'new entrepreneurial spirit' – a spirit characterized by innovation, risk-taking, commitment, and skilled problem solving in the service of a better future.

Achieving Excellence: A Renewed Vision for Education in Ontario, April 2014, p. 4

In order for students to achieve excellence in an area like mathematics, there must be a balance between understanding the basic math concepts, practising skills, and developing the thinking skills needed for advanced problem solving. Learners need to develop characteristics such as perseverance, resilience, and imaginative thinking to overcome challenges.

Achieving Excellence: A Renewed Vision for Education in Ontario, April 2014, p. 5

SETTING THE CONTEXT: Provincial

The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society. To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that help them develop mathematical understanding; learn important facts, skills, and procedures; develop the ability to apply the processes of mathematics; and acquire a positive attitude towards mathematics. The Ontario mathematics curriculum for Grades 1 to 8 provides the framework needed to meet these goals.

The Ontario Curriculum Grades 1 – 8 Mathematics Revised (2005), p. 3

The mathematical processes cannot be separated from the knowledge and skills that students acquire throughout the year. Students must problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of concepts, and the skills required in all the strands in every grade.

The Ontario Curriculum Grades 1 – 8 Mathematics Revised (2005), p.11

SETTING THE CONTEXT: York Region District School Board

Math instruction includes a variety of critical thinking and problem-solving strategies that are taught, practiced and consolidated within the context of problem solving. Through problem solving and the investigation of mathematical concepts, students become a part of a math-talk learning community. Students are actively involved in:

- constructing concepts in a variety of ways
- working with concrete materials
- demonstrating mathematical thinking
- using investigation and inquiry to explore problems
- sharing ideas, asking questions, and discussing strategies as co-learners.

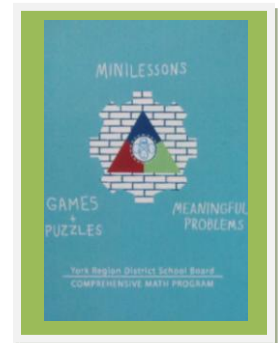
<https://bww.yrdsb.ca/services/cis/mathframe/Pages/default.aspx>

Comprehensive Math Program

<https://bwwyrdsb.ca/services/cis/mathliteracy/Pages/default.aspx>

#YRDSBmath

<http://youtube/SjmoqMpsrEA>



CROSS CASE ANALYSIS

Analysis involving the application of the mathematical processes and their impact on student learning has been informed by individual inquiries conducted by the York Region District School Board (YRDSB) – Student Work Study teacher researcher team.

The mathematical processes cannot be separated from the knowledge and skills that students acquire throughout the year. Students must problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of concepts, and the skills required in all the strands in every grade.

The Ontario Curriculum Grades 1-8 Mathematics Revised (2005), p. 3

| Mathematical Processes | Indicators of Process Application in Joint SWST Studies (Drawn from one or more learning experiences from the studies) |
|---|--|
| <p>Problem Solving</p> <p>Engaging in a task for the solution is not obvious or known in advance. To solve the problem, students must draw on their previous knowledge, try out different strategies, make connections, and reach conclusions. Learning by inquiry or investigation is very natural for young children.</p> <p><i>Guide to Effective Instruction in Mathematics, K-6, Volume 1, p. 98</i></p> | <ul style="list-style-type: none"> • engaging and authentic learning experiences • understanding the problem is critical • activation makes the problem clear • actively involved using appropriate strategies • multiple entry points • meaningful contexts • challenging yet within the proximal zone |
| <p>Communicating</p> <p>Communication is an essential process in learning mathematics. Through communication, students are able to reflect upon and clarify their ideas, their understanding of mathematical relationships, and their mathematical arguments.</p> <p><i>The Ontario Curriculum, Grades 1-8, Mathematics (Revised), p. 17</i></p> | <ul style="list-style-type: none"> • multiple opportunities to talk • vocabulary development – words / symbols • explicit expectations for using math vocabulary • feel safe and comfortable in a math community |
| <p>Reasoning and Proving</p> <p>The process involves exploring phenomena, developing ideas, making mathematical conjectures, and justifying results.</p> | <ul style="list-style-type: none"> • pictorial representations • concrete materials • variety of tools and models used to explain thinking • explanations and proofs incorporated math vocabulary |

| | |
|---|---|
| <p><i>The Ontario Curriculum, Grades 1-8, Mathematics (Revised), p. 14</i></p> | |
| <p>Connecting</p> <p>Seeing the relationships among procedures and concepts helps develop mathematical understanding. The more connections students make, the deeper their understanding. In addition, making connections between the mathematics they learn at school and its applications in their everyday lives not only helps students understand mathematics but also allows them to see how useful and relevant it is in the world beyond the classroom.</p> <p><i>The Ontario Curriculum, Grades 1-8, Mathematics (Revised), p. 16</i></p> | <ul style="list-style-type: none"> • referred to previous classroom experiences to move from known to unknown • transition from concrete to pictorial – then pictorial to abstract – supports development • recognized relationships between numbers supports operational and conceptual understanding • personal connections to build meaningful understanding |
| <p>Reflecting</p> <p>Reflecting on their own thinking and the thinking of others helps students to make important connections and internalize a deeper understanding of the mathematical concepts involved.</p> <p><i>The Ontario Curriculum, Grades 1-8, Mathematics (Revised), p. 14</i></p> | <ul style="list-style-type: none"> • anchors and representations support the reflection process • meaningful learning experiences allow for reflection • self-monitoring and self-checking to confirm reasonableness |

| | |
|--|--|
| <p>Selecting Tools and Computational Strategies</p> <p>Students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.</p> <p><i>The Ontario Curriculum, Grades 1-8, Mathematics (Revised), p. 14</i></p> | <ul style="list-style-type: none"> • access to a variety of concrete tools • experience with a variety of concrete tools and models • tools and models <i>to</i> represent thinking • models <i>for</i> thinking • choice of tools and models by students |
| <p>Representing</p> <p>Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret realistic problem situations.</p> <p><i>The Ontario Curriculum, Grades 1-8, Mathematics (Revised), p. 16</i></p> | <ul style="list-style-type: none"> • concrete, pictorial or numerical organizational tool • students representing their thinking and teacher representing the student thinking • shows relationships between numbers – other mathematical concepts |

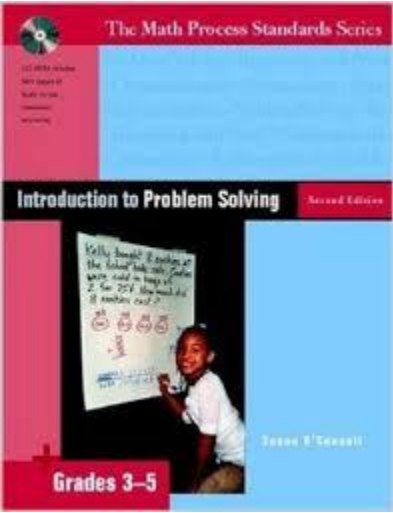
REFLECTION

Based upon what we noticed across the studies, our discussions focused upon the conditions for learning mathematics in relation to:

- shifts in student thinking
- conceptual understanding
- mathematical disposition

As a result, our conclusions contribute to the theory that embedding these processes in mathematical learning experiences enhances students' ability to communicate their thinking and understanding when solving meaningful problems. According to research, "Structured reflection has been shown to be a way to enhance understanding and problem solving." (Eyleer & Giles, 1999 as cited in *Making Thinking Visible*, 2011, p. 13)

RESEARCH SAYS...

| Problem Solving | What Research Says |
|---|--|
| <p data-bbox="94 1102 738 1207">Introduction to Problem Solving, Grades 3-5 of The Math Process Standards Series edited by Susan O'Connell</p>  | <p data-bbox="764 1102 1513 1669">In problem-centered instruction, rather than telling students key math ideas, problems are posed to engage students in exploration and promote thinking about the important mathematical concepts. Students explore problems with partners or groups and are guided in that exploration by the teacher. Students are actively engaged in learning. They are asked to communicate their ideas, share their insights, apply their previously learned knowledge to new situations, reflect on their experiences, and ultimately discover new math ideas. Through problem tasks, new knowledge is built on existing knowledge. In problem-based instruction, the process of learning is as important as the content being learned. Students are learning new ideas but are also learning "how" to learn new ideas. (p. 8)</p> <p data-bbox="764 1675 1513 1858">In today's classrooms, our goal is to help our students use their math knowledge to solve problems rather than to mechanically perform computations. Problem solving is both a goal and a vehicle for our students. (p. 7)</p> |

Introduction to Problem Solving, Grades PreK-2 of The Math Process Standards Series edited by Susan O'Connell

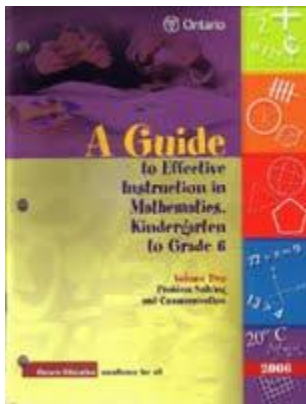


Problem-based instruction is more than simply posting a problem and asking students to solve it. In problem-based instruction, teachers are challenged to support students as they solve problems. They must select appropriate tasks, guide students as they engage in the tasks, facilitate discussions and sharing about solutions, and assess students' understanding of the math content and problem-solving process. Teachers' observations during the problem-solving tasks often result in ideas for related classroom lessons or mini-workshops. These essential teacher responsibilities influence the success of the problem-solving activity.
(p. 10)

Communicating

What Research Says

Guide to Effective Instruction in Mathematics, K-6, Volume 2, Problem Solving and Communication

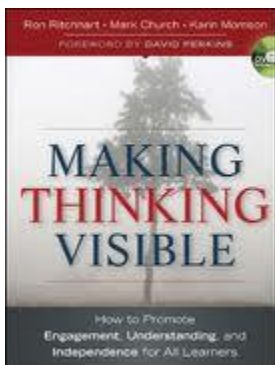


When communication is emphasized in the mathematics program, students also have many opportunities to develop and reinforce their literacy skills. In order to investigate mathematical concepts and solve mathematical problems, students need to read and interpret information, express their thoughts orally and in writing, listen to others, and think critically about ideas. Many of the communication strategies described in this chapter are not unique to mathematics learning – they are instructional techniques that can be used across the curriculum.
(p. 55)

Reasoning and Proving

What Research Says

Making Thinking Visible by Ritchhart, Church & Morrison



The process of understanding is integrally linked to our building explanations and interpretations.
(p. 11)

<http://www.nctm.org/standards/content.aspx?id=26860>

Instructional programs from prekindergarten through grade 12 should enable all students to:

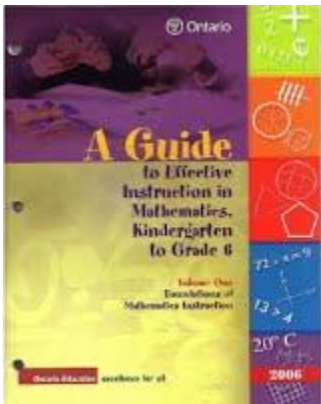
- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Through the use of reasoning, students learn that mathematics makes sense. Reasoning and proof must be a consistent part of students' mathematical experiences in prekindergarten through grade 12. Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts and from the earliest grades.

Connecting

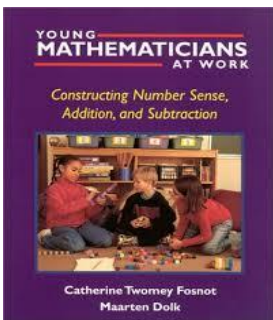
What Research Says

Guide to Effective Instruction in Mathematics, K-6, Volume 1, Foundations of Mathematics Instruction

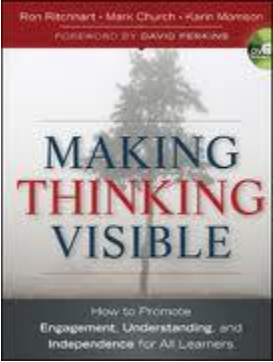
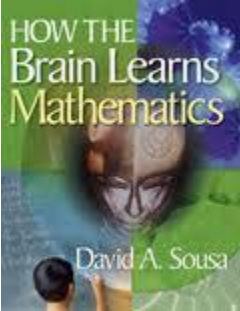


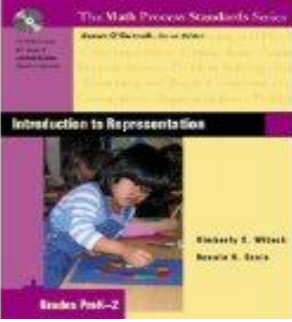
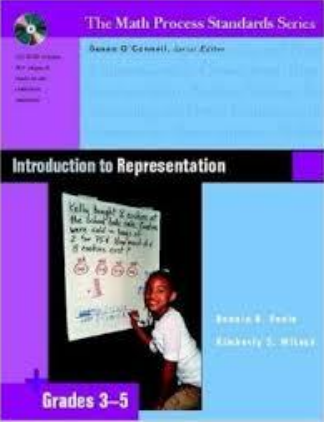
When undertaking a learning task that is developmentally appropriate, a student can use his or her prior knowledge as a mental network in which new ideas and knowledge can be integrated. If the new knowledge “connects”, the student’s thinking is stretched outwards; the student integrates newer, more complex concepts; and optimal learning occurs. The student will have been working in what Lev Vygotsky (1896-1934) calls the “zone of proximal development”. If the learning is too easy (below the zone of proximal development), the student does not gain new knowledge and may become completely disengaged from the process of learning. If the learning is too complex (beyond the zone of proximal development), independent learning does not occur, and the student often experiences frustration and depleted self-confidence. The most meaningful learning occurs within the zone of proximal development.
(p. 29)

Young Mathematicians at Work by Fosnot & Dolk



Many children still connect their model to the situation. Will they be able to use the model in other contexts? How do we help children generalize from a particular situation to all (subtraction and addition) situations? How do we help them generalize the number line model across contexts so that it can be a helpful tool, a model for thinking across contexts – a model to calculate with?
(p. 89)

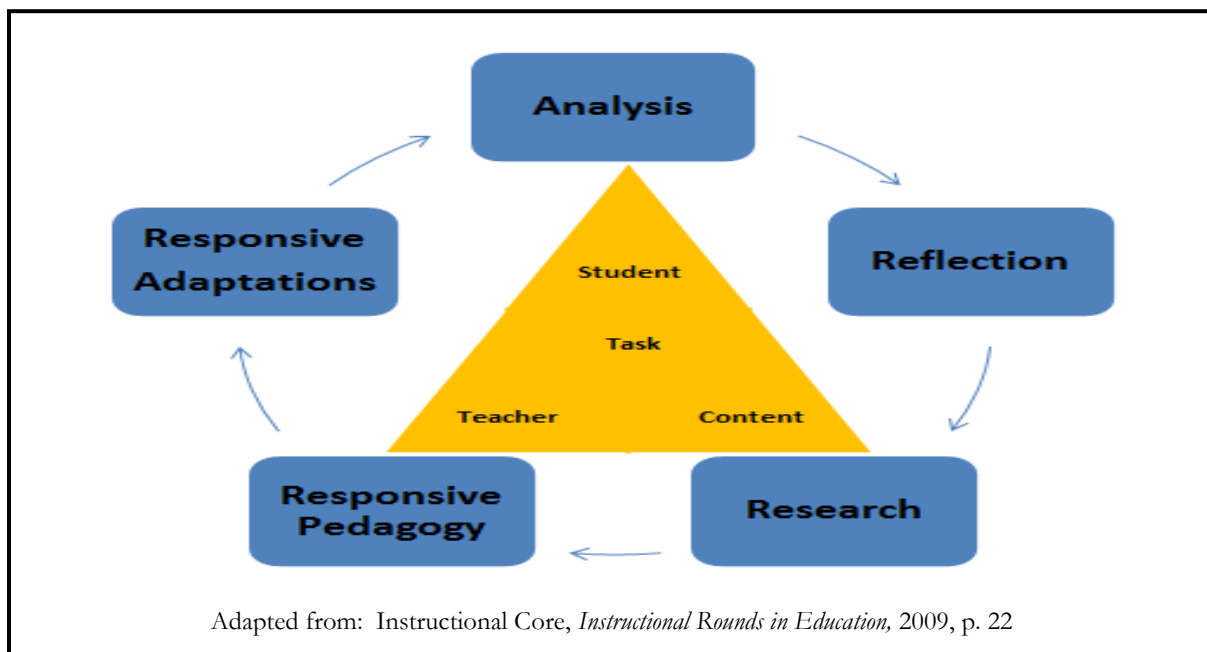
| Reflecting | What Research Says |
|--|---|
| <p data-bbox="94 159 732 226">Making Thinking Visible by Ritchhart, Church & Morrison</p>  | <p data-bbox="764 159 1511 552">Structured reflection has been shown to be a way to enhance understanding and problem solving (Eyleer & Giles, 1999). The answer is that a structured reflection – that is, reflection that goes beyond voicing one’s opinion or feelings – involves describing the object of reflection and noticing its key features, connecting what is new to what one already knows, and examination of the event or object of reflection through various lenses or frames, which is perspective taking. (Colby, Beaumont, Ehrlich, & Corngold, 2009 as cited in Making Thinking Visible, pp. 13 - 14)</p> |
| Selecting Tools and Computational Strategies | What Research Says |
| <p data-bbox="94 835 732 903">How The Brain Learns Mathematics by David Sousa</p> <p data-bbox="94 915 732 982">The Concrete-Pictorial-Abstract Approach - (CPA) Approach</p>  | <p data-bbox="764 835 1511 1192">Concrete components include manipulatives (for example, Cuisenaire rods, foam-rubber pie sections, and markers), measuring tools, or other objects the students can handle during the lesson. Pictorial representations include drawings, diagrams, charts, or graphs that are drawn by the students or are provided for the students to read and interpret. Abstract refers to symbolic representations, such as numbers or letters, that the student writes or interprets to demonstrate understanding of a task. (p. 186)</p> <p data-bbox="764 1230 1511 1913">When using the CPA approach, the sequencing of activities is critical. Activities with concrete materials should come first to impress on students that mathematical operations can be used to solve real-world problems. Pictured relationships show visual representations of the concrete manipulatives and help students visualize mathematical operations during problem solving. It is important here that the teacher explain how the pictorial examples relate to the concrete examples. Finally, formal work with symbols is used to demonstrate how symbols provide a shorter and efficient way to represent numerical operations. Ultimately, students need to reach that final abstract level by using symbols proficiently with many of the mathematical skills they master. However, the meanings of those symbols must be firmly rooted in experiences with real objects. Otherwise, their performance of the symbolic operations will simply be rote repetitions of meaningless memorized procedures. (pp. 186 – 187)</p> |

| Representing | What Research Says |
|---|--|
| <p>Introduction to Representation, Grades PreK-2 of The Math Standards Series edited by Kimberly Witeck & Bonnie Ennis</p>  | <p>Just as students progress from concrete manipulatives to pictorial representations, they then progress to perhaps the most abstract form of representation: numbers and symbols. As students mature in their mathematical understandings, they reach a point where they are ready to learn about the numbers and symbols associated with concepts and processes. Timing is very important: introducing the abstract too early may result in shutting down the meaning- and sense-making processes in children, and they may become overly concerned with memorizing procedures and formulas rather than thinking about the mathematics involved in a situation. An important role of the teacher is to be attuned to each child's level of understanding at the concrete level <i>before</i> attempting to introduce numbers or symbols.</p> <p>(p. 118)</p> |
| <p>Introduction to Representation, Grades 3 - 5 of The Math Standards Series edited by Bonnie Ennis & Kimberly Witeck</p>  | <p>. . . pictures are an important way in which children learn to show their mathematical thinking, and the sophistication of their pictorial representations grows developmentally. The leap between pictures and numbers, however, is not an easy one for some students. Moving from the concrete to the abstract requires a deep understanding of a topic, although we might argue that there are plenty of students who can use numbers successfully without having worked in the concrete first. However, when we ask them to explain their thinking in words, or even to go from the abstract of numbers to creating a concrete model of the process they used, we find that there are holes in their understanding and that they are often performing certain procedures automatically without a foundation of real understanding about why those procedures work or what they mean. Our goal, then, is to move students from pictures to equations when they are developmentally ready and to guide them toward more standard numerical representations when appropriate.</p> <p>(p. 58)</p> |

THEORY OF ACTION

When students engage in solving meaningful problems using the mathematical processes, their ability to communicate their thinking and understanding is enhanced.

IMPLICATIONS



Disposition of Participants in a Math Community

- A Growth Mindset empowers all participants positively
- Co-create and post expectations for all learning experiences in a math community
- Think and learn like mathematicians
- Feel safe and comfortable
- Feel capable and successful
- Build perseverance

Build a Positive Math Talk Environment

- Nurture a safe environment
- Talk like mathematicians
- Share ideas, ask questions and discuss strategies as co-learners
- Co-create and post expectations to support talk

Meaningful Problems & Context

- Use investigation and inquiry to explore meaningful problems
- Learning experiences are challenging and developmentally appropriate (as per The Ontario Curriculum , Grades 1-8, Mathematics (Revised), 2005)
- Opportunities for students to make thinking visible, communicate understanding and reflect
- Learning is anchored from context
- Manipulatives, tools and models are accessible
- Construct models *of* and *for* thinking
- Big Ideas, Strategies and Models are developed in context and posted for reference

CONCLUSIONS

Based upon cross case analysis and research conducted in support of collaborative teacher inquiries, conclusions contribute to the theory *when students engage in solving meaningful problems using the mathematical processes, their ability to communicate their thinking and understanding is enhanced.*

Implications of this finding demand teacher and student attention to the mathematical processes found in the Ontario Mathematics Curriculum document. Embedding these processes throughout the mathematics program allows students to demonstrate and articulate their solutions when solving problems.

Collective analysis, research and co-learning have revealed three key areas needing to be addressed:

1. Establishment of a safe ‘talk’ learning community;
2. Intentional planning involving integration of the mathematical processes and their application through authentic problems / tasks;
3. Deep understanding of the curriculum expectations integrating big ideas, process expectations and relevant skills.

Conditions must be established enabling students to do the mathematics while developing as young mathematicians.

To create a setting where students are “doing mathematics” means making a shift in the way that tasks are presented to students and how classrooms are organized for mathematics lessons. Doing mathematics begins with posing worthwhile tasks, then creating a risk-taking environment where students share and defend mathematical ideas.

Elementary and Middle School Mathematics Teaching Developmentally 3rd Canadian Edition, Van de Walle et al., 2011, p. 11

The mathematical processes are as much about the mathematics as the expectations themselves and should be experienced simultaneously. Ensuring that expectations and mathematical processes are planned together provides students with opportunities to communicate their thinking and understanding when solving meaningful problems and leads to deeper reflections.

To develop understanding of a subject area, one has to engage in authentic intellectual activity. That means solving problems, making decisions and developing new understanding using the methods and tools of the discipline. We need to be aware of the kinds of thinking that are important for scientist (making and testing hypotheses, observing closely, building explanations . . .), mathematicians (looking for patterns, making conjectures, forming generalizations, constructing arguments . . .), . . . and so on, and make these kinds of thinking the center of the opportunities we create for students.

Making Thinking Visible, Ron Ritchhart, Mark Church, Karin Morrison, 2011, pp. 10-11

It is important that planning include problem-based learning experiences enabling students to engage in critical thinking (intellectual activity) through the inquiry process. To that end, students need:

To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that help them develop mathematical understanding, learn important facts, skills, and procedures, develop the ability to apply the processes of mathematics; and acquire a positive attitude towards mathematics.

The Ontario Curriculum, Grades 1 – 8: Mathematics (Revised), 2005, p.3

In order to plan effectively, educators must have a solid understanding of the curriculum document including both strand and mathematical process expectations, and weave them together to create a balanced, relevant program that meets the needs of students.

When developing their mathematics program and units of study from this document, teachers are expected to weave together related expectations from different strands, as well as the relevant mathematical process expectations, in order to create an overall program that integrates and balances concept development, skill acquisition, the use of processes, and application.

The Ontario Curriculum, Grades 1 – 8: Mathematics (Revised), 2005, p. 7

In conclusion, through the processes involved in collaborative teacher inquiries, we have come to understand that:

Meaningful change results when effective, research-based, instructional strategies are used regularly by all classroom mathematics teachers, understood and emphasized by mathematics leaders, and supported by school administrators.

Visible Thinking in K – 8 Mathematics Classroom, Ted Hull, Don Balka & Ruth Harbin Miles, 2011, p. viii



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